Anomaly Detection

Dimensionality reduction: Autoencoders

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Outline

- Autoencoder architecture
- Convolutional autoencoders (CAE)
- Sparse autoencoder (SAE)
- Variational autoencoders (VAE)
- Connections with other methods

The course main references are Kramer 1991 and Dumoulin and Visin 2016.

Preliminaries

Dimensionality Reduction

Convolution

Neural Networks

Backpropagation

Autoencoders

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Remark: this setting can be solved by any dimensionality reduction method (ex. EVD, SVD, PCA, R-PCA etc.).

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Remark: anomalies are the data whose reconstruction error $\|x - x'\|$ is high due to the new subspace.

AE: Architecture

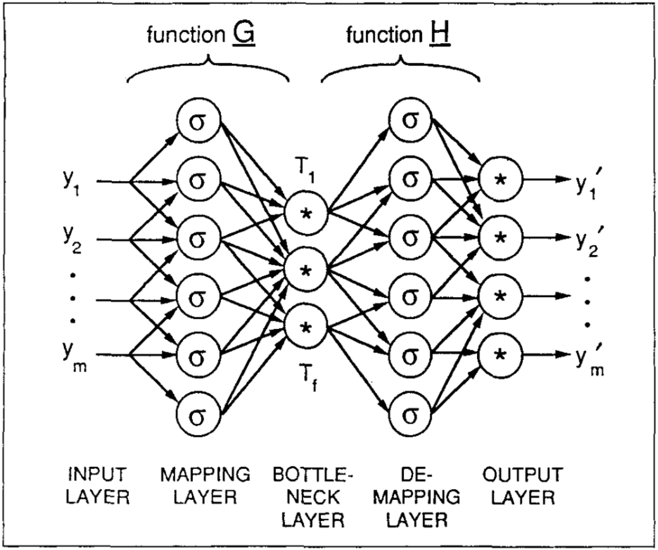


Figure: Autoencoder architecture proposed in (Kramer 1991)

Convolutional Autoencoders

Convolutional Autoencoders (CAE)

Convolutional Autoencoders use convolutional and pool layers in the neural network architecture.

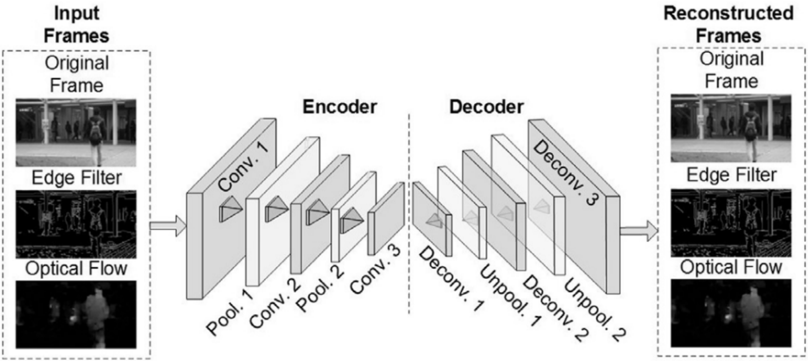


Figure: CAE for anomaly detection in videos (Ribeiro, Lazzaretti, and Lopes 2018)

Convolution Padding

Replication Padding

5	5	0	8	7	8	1	1
5	**5**	**0**	**8**	**7**	**8**	**1**	1
1	**1**	**9**	**5**	**0**	**7**	**7**	7
6	**6**	**0**	**2**	**4**	**6**	**6**	6
9	**9**	**7**	**6**	**6**	**8**	**4**	4
8	**8**	**3**	**8**	**5**	**1**	**3**	3
7	**7**	**2**	**7**	**0**	**1**	**0**	0
7	7	2	7	0	1	0	0

Reflection Padding

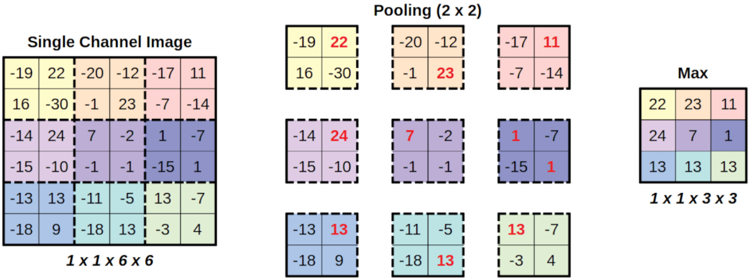
9	1	9	5	0	7	7	7
0	**5**	**0**	**8**	**7**	**8**	**1**	8
9	**1**	**9**	**5**	**0**	**7**	**7**	7
0	**6**	**0**	**2**	**4**	**6**	**6**	6
7	**9**	**7**	**6**	**6**	**8**	**4**	8
3	**8**	**3**	**8**	**5**	**1**	**3**	1
2	**7**	**2**	**7**	**0**	**1**	**0**	1
3	8	3	8	5	1	3	1

Circular Padding

0	7	2	7	0	1	0	7
1	**5**	**0**	**8**	**7**	**8**	**1**	5
7	**1**	**9**	**5**	**0**	**7**	**7**	1
6	**6**	**0**	**2**	**4**	**6**	**6**	6
4	**9**	**7**	**6**	**6**	**8**	**4**	9
3	**8**	**3**	**8**	**5**	**1**	**3**	8
0	**7**	**2**	**7**	**0**	**1**	**0**	7
1	5	0	8	7	8	1	5

Source: https://en.wikipedia.org/wiki/Pooling_layer

Convolution Max Pooling



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CAE: Average Pooling

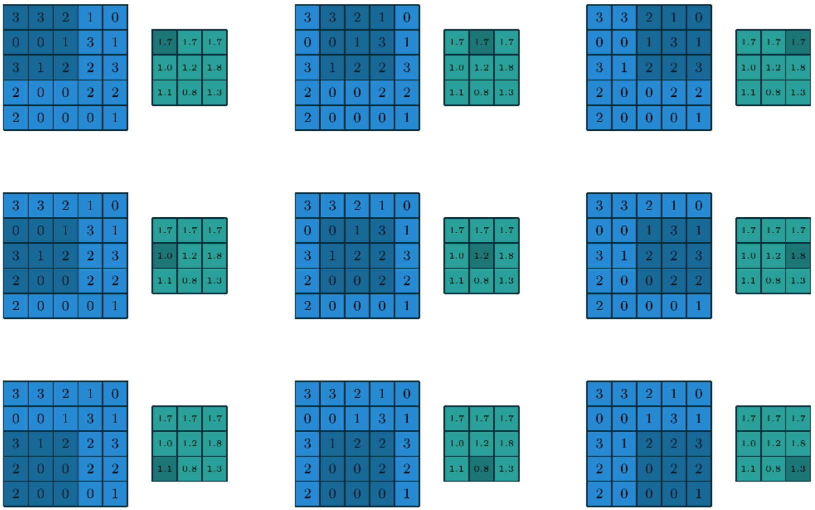

Figure: CAE with average pooling (Dumoulin and Visin 2016)

CAE: Max Pooling

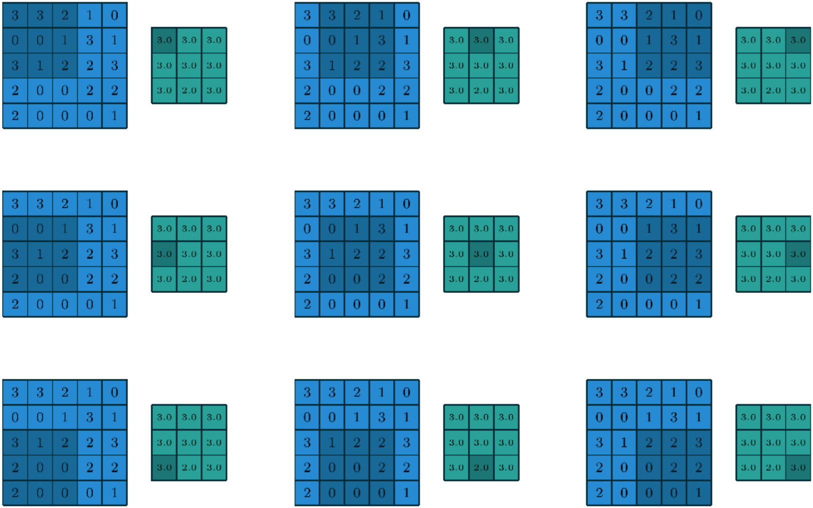

Figure: CAE with max pooling (Dumoulin and Visin 2016)

CAE: Deconvolution is Transposed Convolution

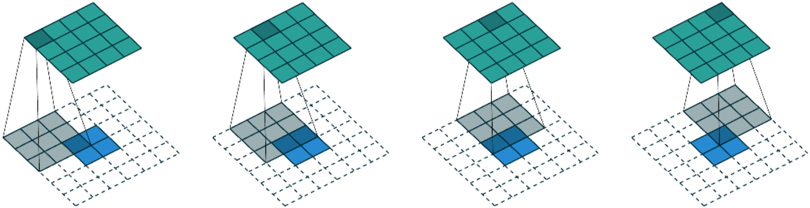

Figure: CAE with transposed convolution (Dumoulin and Visin 2016)

Remark: this includes a convolution and upscaling operation that are sometimes termed "deconv" and "unpool".

CAE: Transposed Convolution with Stride

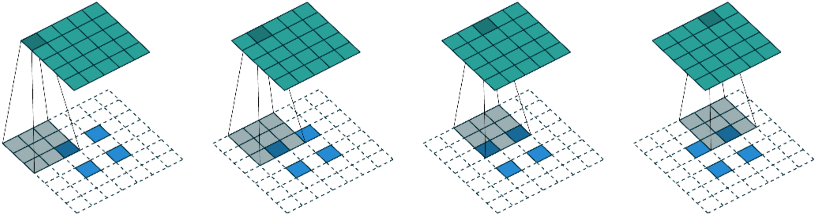

Figure: CAE transposed convolution with stride (Dumoulin and Visin 2016)

Sparse Autoencoders

Sparse Autoencoders (SAE)

Sparse autoencoders enforce sparsity in the mid-layer representations thus allowing subspaces of size $\tilde{m} > m$ but with at most $k < m < \tilde{m}$ non-zeros.

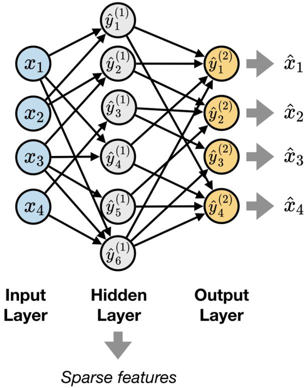

Figure: SAE with overcomplete layer (Dutta et al. 2018)

SAE: Hard-Thresholding with ℓ_0

k-Spare Autoencoders Makhzani and Frey 2013 compute the activation function in each node and then select the k largest values

$$x^{(\ell)} = \mathrm{max_k}\, \varphi_{\omega_\ell}(x^{(\ell-1)}) \tag{2}$$

where $\mathrm{max_k}$ is the function selecting the top-k values in x whose associated support is $\mathrm{supp_k}$.

SAE: Regularization with ℓ_1

Sparse regularized autoencoders add regularization to the AE problem

$$\arg\min_{\omega_E,\omega_D} \|X - D_{\omega_D}(E_{\omega_E}(X))\| + \lambda\phi(X) \tag{3}$$

where $\phi(X)$ is a sparse inducing penalty usually applied at each layer.

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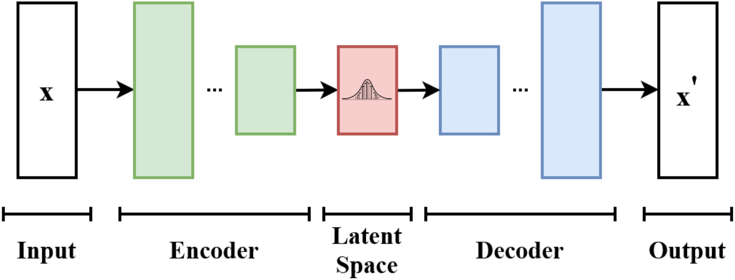
The ℓ_1 convex norm is a popular candidate for $\phi(\cdot)$ such that at each layer we have

$$\phi(X) = \sum_{\ell} \omega_\ell \left\| \varphi(X^{\ell-1}) \right\|_1 \tag{4}$$

Variational Autoencoder (VAE)

Variational autoencoders (Kingma 2013) model input vector x as a mixture of (Gaussian) distributions.



Source: https://en.wikipedia.org/wiki/Variational_autoencoder

AE: Subspace Connections

The resulting autoencoder subspace representations z or final representations x' are coupled with existing methods to enhance anomaly detection performance.

DeepSVDD: AE and SVDD

Given dataset X and autoencoder (E, D) with weights ω, DeepSVDD (Ruff et al. 2018) formulates the soft-boundary as

$$\min_{R,\omega} R^2 + \frac{1}{\nu N} \sum_{i=1}^{N} \max\{0, \|D(E(X)) - c\|^2 - R^2\} + \frac{\lambda}{2} \sum_{\ell} \left\|\omega^{\ell}\right\|_F^2 \quad (5)$$

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For unbalanced datasets, where we have more normal data than anomalies, the authors propose a simplified one-class objective

$$\min_{\omega} \frac{1}{N} \sum_{i=1}^{N} \|D(E(X)) - c\|^2 + \frac{\lambda}{2} \sum_{\ell} \left\|\omega^{\ell}\right\|_F^2 \quad (6)$$

DeepOCSVM: AE and OC-SVM

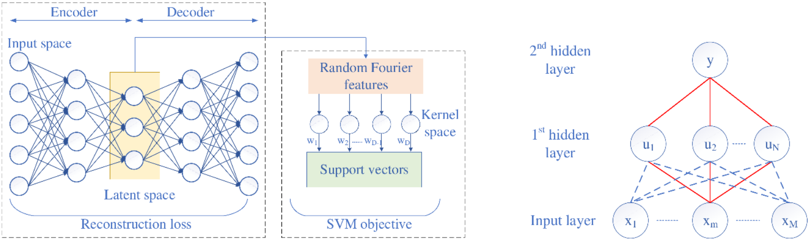

Figure: DeepOCSVM architecture with Fourier Features (Nguyen and Vien 2019)

The DeepOCSVM loss is

$$\alpha \|X - D(E(X))\| + \frac{1}{2}\|w\|^2 - \rho + \frac{1}{\nu N}\sum_{i=1}^{N} \max\{0, \rho - w^\top z(x_i)\} \quad (7)$$

where $z(\cdot)$ is the Random Fourier Features (RFF) function.

References

Dumoulin, Vincent and Francesco Visin (2016). “A guide to convolution arithmetic for deep learning”. In: *arXiv preprint arXiv:1603.07285*.

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